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Numerical analysis of the vertical vibrations of rolling mills and their negative effect on the sheet quality

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Abstract

Vibrations in the frequency range (120–180 Hz) are among the phenomena causing the most critical profile defects of the final product (metal sheet). These defects are caused by the backward self- and parametrically excited movement of the working rolls. The equation of motion of the strip has been derived in taking into account the transportation motion with an average feed velocity.

It is demonstrated that the strip feed velocity plays a crucial role in the excitation and intensity of vibrations. A broader picture of the phenomenon has been obtained compared to the results discussed in the earlier literature. It has turned out that apart from vibrations taking place at a constant amplitude, the beating and chaotic oscillations can get excited. At high strip feed velocities an unstable increase of vibrations can take place, resulting in the rupture of the strip. The velocity–frequency characteristics have been considered, taking the maximum vertical displacements of the roller axes as the measure of the excited vibrations.

Keywords: Self excited-parametric vibrations; Rolling stand; Strip

1. Introduction

The vibrations of the subassemblies of the rolling stands and the strip in the frequency range (120–180 Hz) are among the phenomena causing the most critical profile defects of the metal sheet which is the final product of the rolling process. These defects are caused by the backward self- and parametrically excited movement in the vertical plane of the working rolls [5,6]. The paper includes the analysis of the influence of the parameters of the rolling stand, strip and the rolling process on the intensity of vibrations being excited and in consequence on the level of the strip thickness unevenness relative to its nominal thickness.

2. Assumptions and model description

Earlier modal analysis of the classical discrete-continuous model indicates that for vibrations with the second mode shape (for frequencies 120–150 Hz) the roller pairs (consisting of the working and backing rolls) behave as rigidly connected lump masses [1], and so a lumped-parameter model of the rolling stand is used in the present paper. Using the approach based on the stochastic identification method discussed in [3] the model has been reduced to one with 2×1 degrees-of-freedom. The action of the strip on the rolls has been described using the results of the investigations by Świątoniowski [4] of the distribution of normal and shear stresses in the plasticized part of the sheet.

The most important results of this approach are shown in Fig. 1.

Symbols in Fig. 1:

$$t_x = 3\eta r \frac{d\varphi}{dt}\left(1 - \frac{z_n}{z}\right)\frac{1}{z}$$

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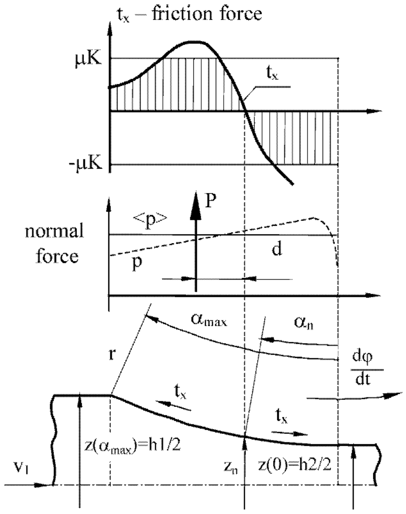

Fig. 1. Normal and tangential stress distributions on the roll-material contact surface.

η is the coefficient of viscosity, z_n the half thickness of the neutral cross section, and b the width of the strip

$$\Delta h = h_1 - h_2$$

P is the force of vertical action of the strip on the working roll, p the normal stress, v_1 the strip velocity at the gap entrance, and v_2 the average strip velocity at the gap exit.

The stress distributions have been obtained assuming an equal angular velocity of both rolls and the non-slip condition. In Fig. 1 the neutral cross-section z_n is the vertical section of the strip for which all its points have the same velocity, approximately equal to the circumferential velocity of the rolls $\frac{d\varphi}{dt}$:

$$z_n = \frac{h_1 v_1}{\omega r}$$

Fig. 2 shows the reaction forces of the strip in the roll bite, neglecting the inertial and dissipative forces. These forces are equal to:

$$T_x = br \int_0^{\alpha_m} t_x \, d\alpha,$$

$$N_x = br \int_0^{\alpha_m} p \sin\alpha \, d\alpha \cong br \int_0^{\alpha_m} p\alpha \, d\alpha,$$

$$N_y = br \int_0^{\alpha_m} p \cos\alpha \, d\alpha \cong br \int_0^{\alpha_m} p \, d\alpha \qquad (1)$$

where $\alpha_m = \alpha_{max}$, and the remaining symbols are shown in Fig. 1. The tensile force acting on the free part of the strip on the side where the strip exits the roller stand is thus given by the following expression:

$$N_2 = N_1 + 2br \int_0^{\alpha_m} [p\alpha - t_x] \, d\alpha \qquad (2)$$

Introducing the formula for the tangential force shown in Fig. 1 into Eq. (2) and performing the integration, one arrives

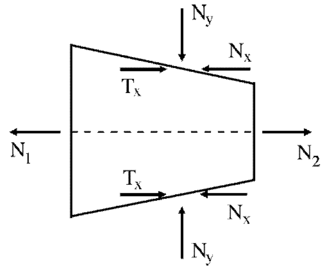

Fig. 2. Equilibrium equations of the strip element in the roll bite. N_1, N_2: front and back tension; T_x: friction force, N_x, N_y: components of the roll reparating force.

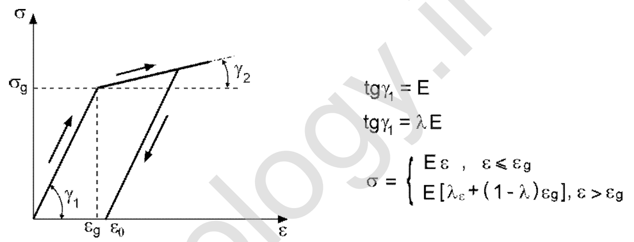

Fig. 3. Transformed model of the stress-strain curve: ε_g: deformation at the yield point.

at the following expression for the T_x component of the force:

$$T_x = \frac{6\eta br}{\gamma(a-1)} \left\{ \frac{z_{n0} + a + 1}{a+1} \operatorname{arc} tg \left[\gamma tg \frac{\alpha_m}{2} \right] - z_{n0} \sqrt{\frac{a+1}{(a-1)^3}} tg \left(\frac{\alpha_m}{2} \right) \left[1 + \frac{(a-1)^2}{a+1} \frac{1}{1 + \gamma^2 tg^2 \frac{\alpha_m}{2}} \right] \right\} \qquad (3)$$

The following symbols have been used in Eq. (3):

$$z_{n0} = \frac{z_n}{r}, \qquad a = 1 + \frac{z(0)}{r}, \qquad \gamma^2 = \frac{a+1}{a-1} \qquad (4)$$

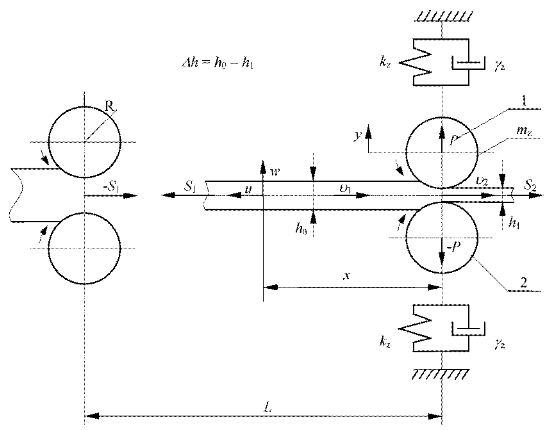

Fig. 4. Physical model of the system rolling stand–strip; v_1, v_2: velocities at the input and output of the gap; S_1, S_2: axial forces in the free sections of the strip, $u(x,t)$, $w(x, t)$: the co-ordinates describing the position of the points on the strip, m_z, k_z, γ: mass, stiffness and equivalent damping coefficient of the rolling stand upper part (the same parameters characterize its lower part).

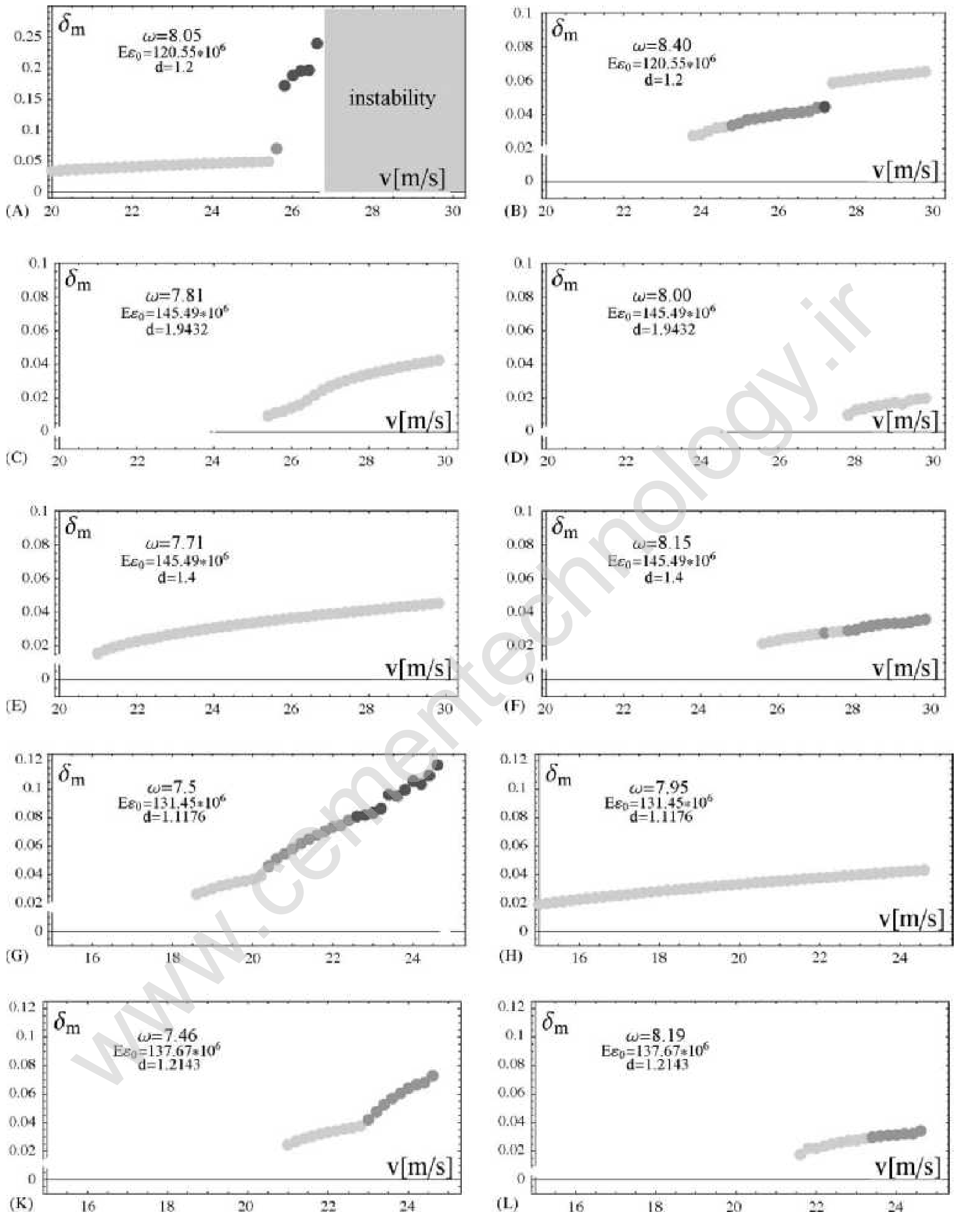

Fig. 5. Maximum changes of strip thickness δ_m, taken relative to the initial strip thickness h_1 vs. strip feed velocity: $\omega = \omega_k/\omega_s^1$, ω_k: rolling stand second natural frequency, $\omega_s^1 = 99.56$ 1/s: first natural frequency of the strip. Points show light grey: steady-state, regular, dark grey: chaotic vibrations. Dark regions mark unstable behaviour.

Assuming that the vertical movement of the rolls is characterized by the generalized co-ordinate y one has:

$$z_n = \langle z_n \rangle + y, z(0) = \langle h_2 \rangle + y \quad (5)$$

where $\langle z_n \rangle$, $\langle h_2 \rangle$ stand for the average values of the parameters, calculated for the steady-state operation regime of the rolling mill.

In order to find the explicit forms of functions describing N_x and N_y (Fig. 2) a hypothesis has been formulated which describes the stress-strain relation in the elastic–plastic region. According to this hypothesis, the real draft characteristic which is described by a continuous curve is replaced by a piecewise linear shape, as shown in Fig. 3.The parameters λ and ε_r should be selected in a way ensuring that the model is compatible with the deformation process taking place in a rolling stand. The residual strain ε_r denotes the relative draft in the lower stand equal to $\frac{\Delta h}{h_1}$ (Fig. 1) and the value of λ should be selected so that the component N_y is approximately equal to (within the limits set by measurement accuracy) the average force within the set screws, during the steady-state motion of the strip.

This parameter (which is characteristic of each rolling stand and depends on the parameters of the rolling process) can be calculated from the following non-linear equation (which is given here without going into detail of its lengthy derivation):

$$\frac{2}{3}\left(\frac{h_1}{R_w}\right)^{1/2} \varepsilon_r \lambda (1-\lambda)^{-3/2} - \frac{P_0}{EbR_w} = 0 \quad (6)$$

The knowledge of parameter λ and the function $T_x(\xi)$ determined by Eq. (3) allows the calculation of the strip axial stress N_2 as a function of variable $\xi = \frac{y}{h_1}$ describing the vertical movement of the roll axes.

Disregarding the tangential component which results from the presence of technological tension, the surplus dynamic force $\Delta N(\xi)$ brought about by the vertical movement of the rolls is equal to:

$$\Delta N(\xi) \cong \{T_x(0) + (1-\lambda)Ebh_1\varepsilon_g\}\xi + \frac{1}{2}T_x''(0)\xi^2 \quad (7)$$

This pulsating force can excite under some conditions a transverse motion of the strip, which will sustain the self-excited oscillations of the rolling stand. It is to be noted that relation (7) is non-linear, even for the assumed piecewise linear physical model. Additionally, this result has been obtained using the model, which does not include energy dissipation terms, which in the present case are negligible.

The interaction of the yielded part of the strip has been described using the formula of Sims, modified to include terms describing the non-linear dissipative properties

$$P = (\bar{K} - \sigma_0) l \sqrt{R(\Delta h + 2y)} + \alpha_1 \dot{y} + \alpha_2 \dot{y}^3 \quad (8)$$

where $\bar{K}$ is the average yield stress, l the width of the strip, α_1, α_2 the damping coefficients, σ_0 the axial stress.

The last parameters are shown in Fig. 4.The physical model of the system "rolling stand-strip" used in the derivation of the governing equations of the problem is shown in Fig. 4.

The partial differential equations of motion of the strip and the ordinary differential equations of the vertical motion of the rolls are coupled by (7) and (8). Using the Galerkin method the first of these equations can be reduced to an ordinary differential equation. Introducing dimensionless variables and parameters, the system of governing equations can be written in the following form:

$$\begin{aligned}&\ddot{f}_n(\tau) + 2\zeta_1 \dot{f}_n(\tau) + n^2[1 - v_0^2 - a\dot{g}_n(\tau)] f_n(\tau)\\&+ n^4 b f_n^3(\tau) = 0, \qquad \ddot{g}_n(\tau) + 2\omega_n\zeta_2\dot{g}_n(\tau) + 2\omega_n^3\zeta_3\dot{g}_n^3(\tau)\\&+ \frac{\mu}{d(\chi+1)}\omega_n^2 (4bf_n^3(\tau) - \frac{1}{2}a\dot{g}_n(\tau))(1 - dg_n(\tau)) = 0 \end{aligned} \quad (9)$$

where n is the number of half-waves which form along the strip, $f_n(\tau) = \frac{f_n(t)}{f_0}$ where $F_n(t)$ is the function describing the strip vibration at point corresponding to the wave peak, f_0 a constant taken to be equal to 0.01 mm, $g_n(\tau) = \frac{2y}{h_2}$ where h_2 is the strip thickness at output, in the absence of vibrations, $\tau = \omega_0 t$ the non-dimensional time, $\omega_0^2 = \frac{\pi^2 E\varepsilon_0}{\rho L^2}$ where L is the length of the free section of the strip, $\mu = \frac{S(\varepsilon_0)}{K}$ where the meaning is explained by Fig. 4 and Eq. (8), $\chi = \frac{k_s}{k_z}$ where k_s, k_z are the equivalent stiffness of the strip and stator with the set screw, $a = \frac{v_1\omega_0}{L\varepsilon_0\omega_k^2}$ where ω_k is the second frequency of the mill, $b = \frac{\pi^2 f_0^2}{L^2\varepsilon_0}$, $d = \frac{h_2}{2\Delta h}$, $\omega_n = \frac{\omega_k}{\omega_0}$.

The equation of motion of the strip has been derived in [2] including its inertia and taking into account the transportation motion with the average strip feed velocity.

3. Numerical results

The differential equations of the problem have been solved numerically using Mathematica 4.2, and the results have been verified by seventh order R–K method. As a result of numerical calculations it was possible to obtain the dependence of the strip unevenness for different parameters of the rolling stand, strip and the rolling process. The influence of the relative strip elongation ε_{0i}, the rigidity of the rolling stand specified by its second natural frequency ω_{ki}, draft Δh_i and the input strip thickness h_1 have been studied. The results of the numerical analysis are shown in Fig. 5.

4. Conclusions

The presence of self excited – parametric vertical vibrations of the working rolls causes important dimensional defects of the strip thickness – δ_m. The analytical solutions confirm – in agreement with the results of earlier studies [1,2,4] – that such vibrations take place when the respective natural frequency of the strip (usually the fourth one) is approximately equal to half the natural frequency of the vertical vibrations of the rolling stand. The results of the numerical analysis shown in Fig. 5 justify the following conclusions:

- the intensity of mid-frequency oscillations strongly depends on the strip feed velocity. Therefore, such vibrations can be observed at the final stands of the rolling train, where this velocity is greater than 20 m/s;
- rolling stands with a lower vertical stiffness are more susceptible to vibrations (Fig. 5A, C, E, G and K);
- increasing the magnitude of the static tension of the strip stabilises the system rolling stand–strip;
- the realisation of the rolling process with higher relative drafts $\Delta h/h_1$ (rolling reductions); intensifies the phenomenon in a very pronounced way;
- the rolling process with velocities higher than about 25 m/s can lead to long-standing intensive vibrations of unsteady character, causing strip thickness changes as high as 25% of the nominal thickness;
- for relatively low strip tensions and low rolling stand stiffness, vibrations can get excited with amplitudes increasing to infinity, leading to the rupture of the strip Fig. 5A;
- in a relatively broad parameter range, chaotic vibrations appear, which for high feed velocities can change into infinitely increasing type even with very small changes of parameters.

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