Experimental investigation of residual stresses in roller bent wide flange steel sections

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1. Introduction

A structural steel member is most often not initially stress free, but has a set of residual stresses induced during its manufacturing process. Residual stresses are stresses which exist due to uneven cooling or to cold forming and are defined as internal stresses in an externally unloaded member and are therefore in internal equilibrium at any cross-section. Residual stresses are in general of primary importance for structures prone to instability, since the presence of residual stress causes premature yielding and thereby loss of stiffness. When simulating the structural behavior of an element by means of numerical methods it is necessary to incorporate a proper residual stress distribution in order to obtain accurate results.

1.1. Straight hot rolled sections

The residual stress distributions in straight hot rolled wide flange sections are well documented and based on numerous experiments as presented by Huber and Beedle [1], Beedle and Tall [2]. Jez-Gala [3], Mas and Massonet [4], Lay and Ward [5], Daddi and Mazzolani [6], Young [7] and Galambos [8] amongst others. The residual stress distribution in straight hot rolled sections is characterized by compressive residual stresses (−) at the flange tips and tension at the web to flange junctions (+). The web is under compressive and tensile residual stresses, as illustrated in Fig. 1. The magnitude of the residual stresses in these hot rolled shapes greatly depends on the geometric properties and the cooling conditions. When a member is straightened (also called rotozed) after leaving the steel mill to fulfill the straightness requirements of hot rolled shapes, it exhibits a different residual stress distribution as reported by Alpsten [9]. The residual stresses caused by earlier cooling are redistributed depending on the amount of straightening (Fig. 1(c)).

1.2. Cold bent sections

Residual stresses have been measured in cold bent sections earlier. Kato and Aoki [10] investigated the residual stress distribution in cold formed circular hollow sections analytically and experimentally. Weng and White [11] and Weng and Pekoz [12] measured residual stresses in press braked plates. Tan et al. [13] evaluated the residual stress in bent metal sheets. To the authors’ knowledge no experimental investigation with respect
to residual stress distributions has been performed on roller bent wide flange sections. Theoretically obtained residual stresses in cold bent sections using a bi-linear material law have been proposed by Timoshenko [14] and are shown in Fig. 2 with: \( \alpha = \) ratio between the plastic and elastic section modulus or shape factor; \( f_y = \) yield stress; \( h = \) height of cross-section; and \( R = \) the radius of the circular arch. When a bar or plate is plastically bent into a certain radius a plastic stress distribution emerges (Fig. 2(b)). After releasing the acting moments, an elastic release or springback of the member takes place, thereby imposing a residual stress distribution on the already present loading stresses (Fig. 2(c)). The result is a stress distribution, which is a summation of the loading and unloading stresses (Fig. 2(d)). The prediction is founded on a uni-axial stress condition and therefore no stress gradient is present along the width of the beam. The theoretical model has found widespread application in the structural analysis of curved steel, King and Brown [15].

### 1.3. Carrying capacity of arched structures

Curved steel is applied in halls, bridges and buildings where residual stresses can play an important role in determining the structural response. Extensive numerical and experimental research has shown that residual stress and initial geometric imperfections have significant influence on the stability of steel arches prone to either in-plane inelastic buckling (Fig. 3(a)) or out-of-plane inelastic buckling (Fig. 3(b)) and therefore need to be considered to investigate the carrying capacity of steel arches, as illustrated by the following examples.

Numerical computations for welded parabolic arches subjected to uniformly distributed vertical loading with thin-walled box sections were performed by Komatsu and Sakimoto [16] to assess the inelastic lateral buckling load thereby taking into account welding residual stresses (Fig. 3(c)) with \( \sigma_{rx} = f_y \) and \( \sigma_{ty} = 0.2f_y \) to \( \sigma_{tc} = 0.4f_y \). A reduction of the load carrying capacity due to the presence of residual stresses of about 20% was found for parabolic mild steel arches. A finite element study on in-plane inelastic buckling carried out by Pi and Trahair [18] showed that by taking into account a distinctive hot rolled residual stress pattern, the load bearing capacity of arches is reduced. The effects of these residual stresses on lateral inelastic buckling were investigated by Pi and Trahair [17] and it was found that the residual stresses reduce the load carrying capacity up to 11% for pin-ended arches subjected to a uniformly distributed load. A reduction of 12% for arches was observed for arches under uniform compression, Pi and Trahair [19]. The observations were all based on a hot rolled residual stress pattern shown in Fig. 3(d). In Fig. 3(d), \( \sigma_{nl} = 0.5f_y \) is the tensile residual stresses at the web–flange junctions, \( \sigma_{tc} = 0.35f_y \) is the compressive residual stress at the flange tips. The midweb residual stress \( \sigma_{tom} \) is determined from the axial equilibrium condition.

### 1.4. Roller bending process

The cold bending of steel elements with the use of rolls, also known as the roller bending process, is a method that is available for curving arches and is used frequently in practice. In the roller bending process a member is placed in the machine and curved between three rolls at ambient temperature (Fig. 4(a)). Because of the three main rolls' pyramid arrangement the roller bending process is sometimes called pyramidal rolling. Permanent curvature in the member is achieved due to movement of the right roller along a fixed prescribed path (Fig. 4(b)) and subsequent rolling of all rolls (Fig. 4(c)) inducing a process of continuous plastic deformations. The member is rolled back and forth on multiple passes until the desired radius is achieved. Due to placement requirements only a part of the total beam length can be bent, leaving straight material on either side of the curved section. At the inside of the top flange (subjected to elongation) a flange support roller is utilized to provide additional restraint and thereby preventing the web from crippling. The roller bending process can be applied about the weak or strong axes of wide flange sections and also allows the forming of non-circular curved beams. However, this investigation is confined to roller bent sections bent about the strong axis into a circular geometry. A detailed description of the roller bending process has been presented by Bjørhovde [20]. The elongated flange and shortened flange are denoted in this paper as top flange and bottom flange respectively and in the subsequent sections results, will be presented according to this notation.
1.5. Motivation for research

Section 1.3 showed that residual stresses are of paramount importance when investigating the load carrying capacity of steel arches. The computed results were based on arches with either welding residual stresses or residual stress due to differential cooling, not on residual stresses due to cold forming. Since the material experienced severe plastic straining during rolling it cannot be assumed that the residual stresses in roller bent sections are identical to their straight hot rolled counterparts. Incorporating a hot rolled stress distribution in numerical form to assess the load carrying capacity of roller bent arches may therefore yield inaccurate results. The numerical computations could also be carried out by implementing the theoretical residual stress pattern in cold forming as suggested by Timoshenko. But his theoretical residual stress distribution is questioned since the true bending process exhibits a complex interaction between rolls and beam, as explained in Section 1.4, which cannot be represented by a uniaxial loading-unloading scheme. The lack of knowledge with respect to residual stresses in roller bent sections in conjunction with inelastic failure of arches was recognized at Eindhoven University of Technology and a research study was set up. The aim of this study is to experimentally investigate the residual stress patterns of roller bent sections. The sectioning method in conjunction with electrical strain gauges was employed to establish the residual stress distributions for 12 curved steel sections, bent to different radii and with different steel grades. Also measurements on 7 straight reference sections were carried out to assess the influence of the roller bending process. Previous measurements of residual stresses on roller bent sections with the hole-drilling method yielded unsatisfactory results, La Poutre [21]. Since possible disturbances caused by the sectioning method were expected based on previous research, additional measurements were carried out to investigate the robustness of the measurement technique. Tension tests on coupons taken from straight reference sections as well as curved ones were conducted to obtain the stress-strain curve and to relate residual stress measurements to the yield stress.

2. Experimental program

Table 1 shows the experimental program. The program comprised small and medium size wide flange sections with different curvature and steel grade. Each arch was curved from straight hot rolled steel. The initial residual stress distribution and base material properties were determined from the straight sections serving as reference sections (Fig. 5). Coupons were milled from straight reference sections and subjected to a uniaxial tension test to obtain the Young's modulus, yield stress \( f_y \) and tensile stress \( f_t \). For gradually yielding material, the 0.2\% offset proof stress was adopted as the yield stress. Extra tests were performed on curved coupons taken from the top flange to assess the increase in yield stress and tensile stress due to cold forming as associated with the roller bending process. An example of a full stress-strain relationship for both S235 and S355 mild steel for a straight and a curved section are presented in Fig. 6. As shown, both the yield stress and the tensile stress increase and the ductility decreases, as expected.

2.1. Specimen preparation

The sectioning method was used to measure residual stresses in roller bent steel arches. The test specimen was saw cut from larger
Steel arches. Electrical strain gauges were applied to the surface of the roller bent wide flange sections. Electrical strain gauges were selected in preference to a mechanical gauge or Whittomore gauge due to their better accuracy and applicability to curved steel. For this investigation small (2 × 6 mm) electric strain gauges manufactured by Tokyo Sokki Kankyjuo Co. Ltd. were used. The arrangement of the strain gauges is shown in Fig. 7(a). To reduce end effects, the test area was a distance of 2.5 times the height of the beam from the ends (see Fig. 7(b)). The number of strain gauges used on each of the specimens is presented in Table 1. Only the longitudinal stresses were measured.

The specimen was clamped in a vise and the transverse saw cut and subsequent longitudinal saw cuts were made with an electrical band saw and hand saw respectively. The influence of heat release from the electrical band saw cuts was suppressed by supplying fluid coolant. Short-circuiting of the electrical strain gauges was prevented by covering the gauges with a protective layer of paraffin. Strain release was recorded during the entire saw cutting procedure. Measurements for strain were recorded until approximately 30 min after the end of the cutting. Strain measurements were converted to stress values by multiplying the strain by the Young's modulus as obtained from the tensile tests on the straight reference sections, thereby assuming elastic release of the strains. Taken that the residual stress distribution can be modeled as linearly varying through the material thickness a distinction was made between membrane and bending stresses. Stress values on opposite sides were averaged to provide the membrane stresses. Two specimens after the sawing procedure are shown in Fig. 8. Less than 1% of the total number of strain gauges showed signs of malfunction during sawing. The lacking measurements were replaced by values obtained from the same specimen through either linear interpolation or using symmetry.

Table 1

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Type</th>
<th>Mechanical properties straight reference sections</th>
<th>Steel grade</th>
<th>Bending radius R (mm)</th>
<th>Number of strain gauges</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>( f_y (N/m^2) ) [400 ] ( f_c (N/m^2) ) 0.56</td>
<td>S235</td>
<td>1910</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>HE 100A</td>
<td>279</td>
<td>S235</td>
<td>2546</td>
<td>40</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>364</td>
<td>S355</td>
<td>1910</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>357</td>
<td>S355</td>
<td>2546</td>
<td>40</td>
</tr>
<tr>
<td>E</td>
<td>HE 360B</td>
<td>297</td>
<td>8000</td>
<td>4000</td>
<td>108</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td>269</td>
<td>8000</td>
<td>4000</td>
<td>108</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td>357</td>
<td>8000</td>
<td>4000</td>
<td>108</td>
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<td>H</td>
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<td></td>
<td>357</td>
<td>8000</td>
<td>4000</td>
<td>108</td>
</tr>
</tbody>
</table>

Fig. 7. Strain gauge distribution and end distance (each dash represents a single strain gauge).

Fig. 6. Stress-strain curves for specimen B (HE 100A, \( R = 2545 \) mm, S235) and E (HE 100A, \( R = 2545 \) mm, S355) taken from the straight reference section (left) and the curved section (right).
2.2. Additional measurements

The slitting of steel can be performed by means of saw cutting or electric discharge machining (EDM). The first method requires little preparation time and is practically not bound by section dimensions but the latter procedure is free of any large temperature influences which possibly disturb the strain measurements. Weng and Pekoz [12] and Abdel-Rahman and Sivakumaran [22]. For comparison specimen A was tested by the saw cutting procedure and also by employing the electric discharging machining technique in order to investigate possible temperature influences from the saw cutting procedure. In addition specimen F was investigated with the saw cutting procedure at three different positions along the curved member in order to review the variation or scatter of the residual stresses and to assess the robustness of the measurement technique. Based on observations from these additional measurements it can be concluded that temperature effects are not of any significant influence with respect to the experimental data, as obtained with the saw cutting procedure. Also it can be stated that the measurement technique is robust. A full review on the possible disturbance caused by the saw cutting procedure and the scatter of residual stresses at various locations along the member is presented in Appendix.

3. Experimental results

3.1. Introduction

Results for all experimental findings are presented. The residual stresses in the flanges and webs of straight beams due to hot rolling of the various steel sections are shown in Figs. 9-11 for HE 100A, HE 360B and IPE 360 sections respectively. The final residual stress distributions across the sections of curved beams as a result of the roller bending process are given in Figs. 12-14 for the HE 100A, HE 360B and IPE 360 sections respectively. The measured data of the roller bent specimens is supplemented with the theoretical distribution of residual stresses in cold bent sections as proposed by Timoshenko [14] (Fig. 2), incorporating the measured yield stress (Table 1) and shape factor of the straight wide flange sections.

3.2. Straight sections

The residual stresses for the straight HE 100A beams are presented in Fig. 9. The results show typical residual stress distributions for the members when compared to earlier presented results by Daddi and Mazzolani [6] and Mazzolani [23]. It can be seen that for smaller sections, the residual stresses are significantly lower when compared to larger sections. It should be noted that for the straight reference section for specimens D, E and F only a limited number of strain gauges were employed. Measured hot rolled residual stress distributions in the straight HE 360B and IPE 360 beams are presented in Figs. 10 and 11 respectively. The observed stress distributions across the webs in compression and tension near the flanges agree very well with theoretical predictions. It is also shown that the hot rolled residual stresses in the larger HE 360B and IPE 360 sections are higher than in the smaller HE 100A sections confirming previous experimental results by Young [7] that the magnitude of hot rolled stresses is dependent on the section size and geometry. The large differences in residual stresses
between the left and right hand sides of the webs near the fillets are taken to be the result of straightening of the beams where after milling rollers are applied near the fillets Bjoerhovde [20].

3.3. Roller bent HE 100A sections

The residual stress distributions due to roller bending of HE 100A beams are given in Fig. 12. The diagrams show that the maximum tensile stresses are in the bottom flanges at the web to flange junction. The webs of the sections mainly display compressive residual stresses. The stress distributions in the sections of steel beams consisting of S355 material are similar to those for the members with steel grade S235. Although the maximum residual stresses are higher for the S355 members, the maximum tensile and compressive stresses are also observed at the web to flange junctions and webs respectively. It is noted that the maximum measured residual stress in specimen A is 353 N/mm². This is 10% above the yield stress and 18% below the tensile stress of the straight beam. It is noted that the maximum measured residual stress in specimen D is 468 N/mm². This is 29% above the yield stress and 17% below the tensile stress of the straight beam.

3.4. Roller bent HE 360B sections

The residual stress distribution for specimen G and specimen H are shown in Fig. 13. It is observed that compressive stresses and tensile stresses are present in the top flange of the S235 and S355 specimen respectively. The residual stresses in the roller bent member with steel grade S355 are distributed with 473 N/mm² in tension at the bottom flange center and -139 N/mm² at the web.

3.5. Roller bent IPE 360 sections

The residual stress distributions due to roller bending of hot rolled IPE 360 beams are shown in Fig. 14. The average values of the residual stresses in the top flange are quite small but larger values were found in the web and bottom flange. The rather large differences between stresses obtained from the outside measurements and inside measurements along a limited width of the flange indicate the presence of local bending in the top flange. This bending, however, has minor influence on the average stress distribution. It can be seen that the residual stresses obtained from both sides of the web are very close, with the exception of specimen K.

4. Discussion

4.1. Residual stresses in roller bent sections

The measured residual stresses in the roller bent beam sections show very similar distributions in the sections of all specimens. The small stresses in the top flanges are almost uniformly distributed along their widths. The stresses in the webs near the top flanges are in general small, tensile or compressive. They increase in compression towards the bottom flange. The stress distributions along the width of the bottom flanges are non-uniform and display large peaks near the junctions with the webs. The measured stresses on both sides of the webs show small differences. This was expected as the beam was roller bent about the major axis, i.e. little or no bending about the minor axis occurred. The larger differences between the stresses on the inside and outside surfaces of the flanges indicate the presence of bending about the major
axes of the individual flanges. The average state of stress in a top flange is a small uniform tension or compression. The differences between inside and outside stresses in the bottom flanges display distributions which are very different from those in the top flanges. The average state of stress in the bottom flanges are not uniform and in all specimens show both tension and compression.

4.2. Comparison to straight sections

Because of the limited data it is difficult to draw conclusions for the HE 100A sections other than that in general the roller bending process introduces tensile stresses into the top and bottom flanges and compressive stresses into the web.

The more detailed and consistent data for the residual stresses in the HE 360B and IPE 360 sections shows the roller bending process introduces changes towards more uniformly distributed stress distributions along the top flanges. The bottom flanges show significantly increased stresses in the extreme fibers but smaller changes in the average stress distributions. The webs of the sections appear to have been subjected to bending with an introduction of tensile stresses into the top half of the web and compressive stresses into the bottom half. This resulted in a reversed stress pattern in the bottom half with maximum values near the bottom flange.

The findings show that the roller bending process has a significant effect on the longitudinal residual stress distribution, i.e. the roller bending process modifies the initial residual stress distribution due to differential cooling and rotorizing. The deformation strains from the roller bending process significantly alter the doubly symmetric hot rolled residual stress distribution resulting in a new monosymmetric residual stress pattern with respect to the minor axis of bending. The measured values show that the residual stresses in roller bent sections are larger compared to their straight counterparts, indicating that premature yielding is likely to be more prevalent in roller bent sections.

4.3. Influence of bending radius on residual stress

The experimental results for the roller bent HE 100A sections in Fig. 12 and IPE 360 sections in Fig. 14 reveal that there is no clear relationship between increasing curvature and residual stress. However, it can be observed that with decreasing bending radii the compressive stresses in the webs increase marginally whilst the stresses in the top and bottom flanges remain largely unchanged.

4.4. Influence of steel grade on residual stress

In general it can be observed that specimens with steel grade S355 reveal residual stress patterns with a slightly larger magnitude compared to specimens with steel grade S235. However, the top flange of the roller bent S235 HE 360B beam in Fig. 13 displays average compression stresses while the stresses for the S355 beam are tensile across this flange. The residual stresses in the web show different distributions for the two grades of steel, i.e. bending about the major axis for S235 and almost uniform compression for S355 steel. The stresses in the bottom flanges display similar distributions but are much larger for the higher grade steel. It could be suggested that in general with increasing steel grade the stress distributions in the flanges increase or move towards tension and the residual stresses in the webs increase or move towards compression.

4.5. Magnitude of residual stress

The measured residual stresses are generally below the yield stress of the straight material, although the yield stress was exceeded by the residual stress values in the bottom flange. At first sight, this observation violates the conversion from measured strain to stress values as stated in Section 2.1, since this conversion
was based on elastic strain release and therefore restricts the magnitude of the residual stress to a maximum value, i.e. the yield stress. However a higher residual stress than the yield stress can be expected as a result of cold working the material during the roller bending process. Therefore a larger elastic strain release can be expected compared to the maximum strain release as governed by the yield stress of the straight material. Additional tensile tests on coupons taken from curved sections support this theory as they showed an increased yield stress (Fig. 6). The experimental findings of Cruise and Gardner [24] showed a similar phenomenon for residual stresses in stainless steel sections.

4.6. Comparison with theory

Residual stresses and their distribution along the flanges and webs of steel sections that are obtained from theoretical models for cold bent beams are different from measured results. Theoretical models assume residual stress distributions in cold bent sections to be the result of uniaxial loading and reloading. In addition, the models do not account for a residual stress gradient along the width of the flanges. The experimentally obtained stresses do show a large stress gradient along the bottom flange and a small gradient or a nearly uniform distribution in the top flange. This clearly indicates a multi-axial stress state rather than a uniaxial stress state after rolling. Also, the theoretically derived stress pattern postulates an anti-symmetric residual stress distribution about the major axis of bending. This anti-symmetry could not be clearly observed in the specimens with the exception of the IPE 360 sections which to some extent display a zigzag pattern as postulated by Timoshenko, see Fig. 2.

5. Conclusions

An experimental investigation of residual stresses in roller bent wide flange sections and straight hot rolled wide flange sections was presented. The strain readings were obtained using the method of sectioning and subsequently converted into stress values by multiplying the strains by the Young’s modulus. Experimental findings show significant differences in residual stress levels and patterns between roller bent wide flange sections and straight hot rolled wide flange sections. The measured residual stresses in the roller bent sections are larger than those in straight
Fig. 13. Residual stress distribution after roller bending of hot rolled HE 360B sections in N/mm².

Fig. 14. Residual stress distribution after roller bending of IPE 360 sections in N/mm².
sections. Therefore, the suggestion to use a residual stress model based on straight hot rolled sections for curved roller bent members is questionable. Also, the residual stress model proposed by Timoshenko [14] and outlined by King and Brown [15] yields stress patterns that are quite different from experimentally obtained stresses. Residual stress gradients along the flange widths were observed in all curved specimens, indicating that a uniaxial stress assumption for loading and reloading is not valid for modeling residual stress distributions in roller bent sections.

6. Recommendations

In the nearby future it is useful to supplement the current experimental data with finite element simulations of the roller bending process. In addition experiments on annealed specimens can be carried out in order to assess the influence of the initial hot rolled residual stress distribution on the final stresses after roller bending.

The stress distributions from this investigation will be implemented in numerical form to assess the load carrying capacity of steel arches manufactured by roller bending and to investigate whether the adoption of a hot rolled residual stress pattern provides conservative or non-conservative results.

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Appendix

A.1. Influence of temperature

The sectioning method has been used extensively to measure residual stresses. Usually, slitting of steel was performed with a slitting saw or band saw. The possible influence of the saw cutting process on residual stress measurements was considered of minor importance. However, Weng and Pekoz [12] and Abdel-Rahman and Sivakumaran [22] stated that the slitting of steel with the conventional saw cutting procedure induces significant external disturbances caused by heating, clamping and vibration. The electrical discharging machining (EDM) technique practically eliminates the aforementioned disturbances. Although the EDM technique provides better surrounding conditions, it requires more preparation time and is less feasible for large wide flange sections. In order to get insight into possible disturbances due to the saw cutting process, specimen A was tested twice. For comparison, both the EDM technique and the saw cutting procedure were used to measure residual stresses in this specimen. The results of both techniques are shown in Fig. 15, whereas the values of the specimen with the saw cutting procedure were presented earlier in Fig. 12, but are repeated here for convenience. In conjunction with the repeatability of measurements presented in the subsequent section, it is seen that both techniques yield almost identical residual stress patterns. Therefore it was concluded that the external disturbances are not significant and the saw cutting procedure could be used for the remaining specimens.

A.2. Repeatability of measurements

Residual stresses were measured at three different locations along a single member as shown in Fig. 16. The results are presented in Fig. 17 for specimen F. The variation or scatter in the residual stress values along the member is quite small. This indicated that for the other specimens, representative values could be obtained by performing measurements at a single location in the roller bent beam. The results at location 1 are identical to Fig. 12 and are shown here again for convenience.

References